United States Department of Agriculture
Soil Conservation Service
Engineering Division

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ROUTING THROUGH TIDE GATES[1]

In coastal areas diking systems associated with submerged culverts and automatic drainage gates are used to prevent inundation of lowland during high tide. Dikes are used to withhold high tides from the lowlands. Culverts or pipe outlets through the dikes are required to discharge runoff from the land side of the dikes. Automatic drainage gates are provided at the downstream end of the pipes to prevent backflow of the tidewater through the outlets during high tide.

This Technical Release has been prepared to give a method of (1) determining the size of outlet required to prevent inundation of land above a stated elevation; or (2) to determine for a selected pipe of standard size the highest elevation at which inundation will occur.

The method does not attempt to predict the elevation of tide vs time nor the design inflow rate, but assumes that this information has been obtained along with the storage characteristics on the land side of the dike and the rating curves of the pipe.

The following assumptions have been made for the analysis given in this discussion.

1. The tide elevation vs time curve is cyclic. The method involves the use of a design tide curve (tide elevation vs time) for the site. It is assumed that this information is available and that the design tide curve is selected by a consideration of extreme heights and variations of tides.

2. The inflow rate from the land side of the dike is constant. Routing a design inflow hydrograph through the tide gates would be of little value because of the impracticability of predicting the time of the beginning of the storm with respect to the stage of the tide. Routing through tide gates in this Technical Release is based on the assumption that the inflow rate is constant. Sound hydrological principles should be used to select the design inflow rate.

3. The outlet pipe is submerged at all times. Rating curves of pipe for the site should be obtained from pipe manufacturers or prepared for various diameters, lengths, and types of material with tide gates attached. The following analysis can be adjusted to include those situations in which the pipe is not submerged at all times.

4. No reverse flow occurs through the outlets; i.e., no flow is assumed to occur from the tide side to the land side.

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[1]This Technical Release was developed and written by Paul D. Doubt and Gerald E. Oman of the Design Section to meet a specific field problem.

5. The tide gate opens or closes at the instant the head on the pipe varies from zero. The analysis can be adjusted for those situations in which this assumption is invalid.

A topographic survey of the area above the dike is required to prepare a storage vs elevation curve.

The analysis presented is one of steady state and is not to be considered as of a transient state. This requires that the gate opens at the same elevation of the tide E_1 in every cycle of the tide curve and, of course, the gate closes at the same elevation of the tide E_2 in each cycle of the tide curve. The cycle of a tide curve will be considered from high tide to high tide. The symbol E_1 will be used to designate the elevation of the tide at which the gate opens in the first cycle and E_3 will be used to designate the elevation of the tide at which the gate opens in the second cycle. (See Fig. 1.) The times at which the gate opens and closes are designated t_1, t_2, and t_3 and have a datum corresponding to high tide. For a given pipe size there is an elevation of the tide E_2 at which the gate closes corresponding to the elevation of the tide E_1 at which the gate opens. On changing the pipe size there is another elevation E_2 corresponding to another elevation E_1. In other words there exists for a given value of E_1 a corresponding value of E_2 and also a corresponding value of size of pipe. The corresponding values of E_1 and E_2 are ascertained in the following paragraphs.

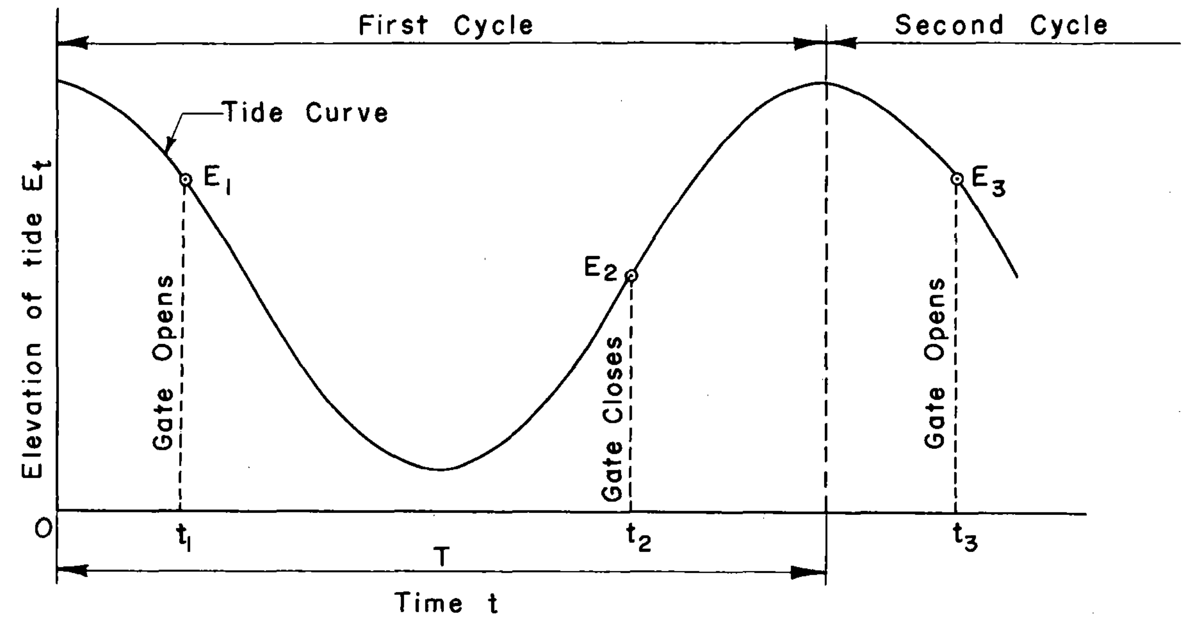

FIGURE 1

Determination of the elevation of the tide E_2 at which the gate closes corresponding to the elevation of the tide E_1 at which the gate opens. The gate closes when the tide elevation and the elevation of the water surface of the storage are both equal to E_2. (See assumption 5.) The gate opens again when the elevation of the tide and the water surface in the storage are again both equal to E_3. During the time interval $(t_3 - t_2)$ the elevation of the tide E_t is greater than the elevation of the water surface E upstream from the dike and the gate is closed and there is no outflow. All of the volume of inflow $(I_3 - I_2)$ that occurs during this time interval is stored in the impounding area upstream from the dike, or

$$I_3 - I_2 = S_3 - S_2 \tag{1}$$

where I_2, I_3 = volume of inflow corresponding to t_2, t_3
S_2, S_3 = volume of storage in the area upstream from the dike corresponding to t_2, t_3

The inflow rate i is constant and during the time interval $(t_3 - t_2)$ that the gate is closed the volume of inflow is

$$I_3 - I_2 = i\ (t_3 - t_2) \tag{2}$$

From Eqs. (1) and (2)

$$S_3 - S_2 = i\ (t_3 - t_2) \tag{3}$$

Equation (3) expresses the relationship between the volume stored during the time interval when no outflow occurs and the time interval itself. Neither the time interval $(t_3 - t_2)$ nor the volume stored $(S_3 - S_2)$ is known. The value E_3 has been selected. The relationship expressed by Eq. (3) is to satisfy the tide curve and the storage curve simultaneously. The difference in elevation $(E_3 - E_2)$ has an associated time interval $(t_3 - t_2)$ according to the tide curve. This difference in elevation according to the storage curve represents a volume of storage which must be equal to the value given by Eq. (3) or $(S_3 - S_2) = i\ (t_3 - t_2)$. The change in elevation $(E_3 - E_2)$ for any value of E_3 can be determined by superimposing a storage time curve drawn on transparent paper over the tide curve. The storage-time curve is obtained from the storage curve by determining the water-surface elevation vs time relationship when the inflow i to the dike is considered constant and the outflow is zero. The coordinates of the storage-time curve are the elevation of the water surface and time and have the same scale as the tide curve. This method of graphical solution for obtaining the correspondence of E_3 and E_2 is illustrated by Step 1 of ES-103, sheet 1. As previously stated the value of E_3 is equal to E_1.

<u>Determination of the approximate value of A_p corresponding to the elevation of the water surface E_1.</u> During the time interval $(t_2 - t_1)$ the elevation of the water surface E upstream from the dike is greater than the elevation of the tide E_t and the gate is open. The rate of discharge is dependent on the value of the discharge head h. The discharge head is the difference between the water-surface elevation and the tide elevation (Fig. 2). In the first cycle the outlet will be discharging water only during the time interval $(t_2 - t_1)$. The pipe is required to discharge the total volume of inflow during one cycle i T in the time interval the gate is open $(t_2 - t_1)$. There is only one outlet size of cross-sectional area A_p that will discharge the volume of inflow i T for a selected value of E_1. For a given value of E_1 there is only one corresponding value of E_2 and only one value of A_p. If any one of these corresponding values is changed the other two are changed. In the solution for outflow through tide gates, simultaneous values of E_1, E_2, and A_p are to be found. The preceding paragraphs give the method of

determining the correspondence of E_1 and E_2. A value of A_p can be found by trial and error by assuming a value of A_p for a selected value of E_1 and routing the flow through the tide gate to determine a routed value of E_2. The routing is then repeated using various values of A_p until the routed value of E_2 is the same as that value of E_2 obtained by the method of the preceding paragraph. The number of times that the routing will have to be repeated can be reduced if the simultaneous values of E_1, E_2, and A_p can first be closely approximated. The method of approximating the value of A_p is given in the following paragraphs.

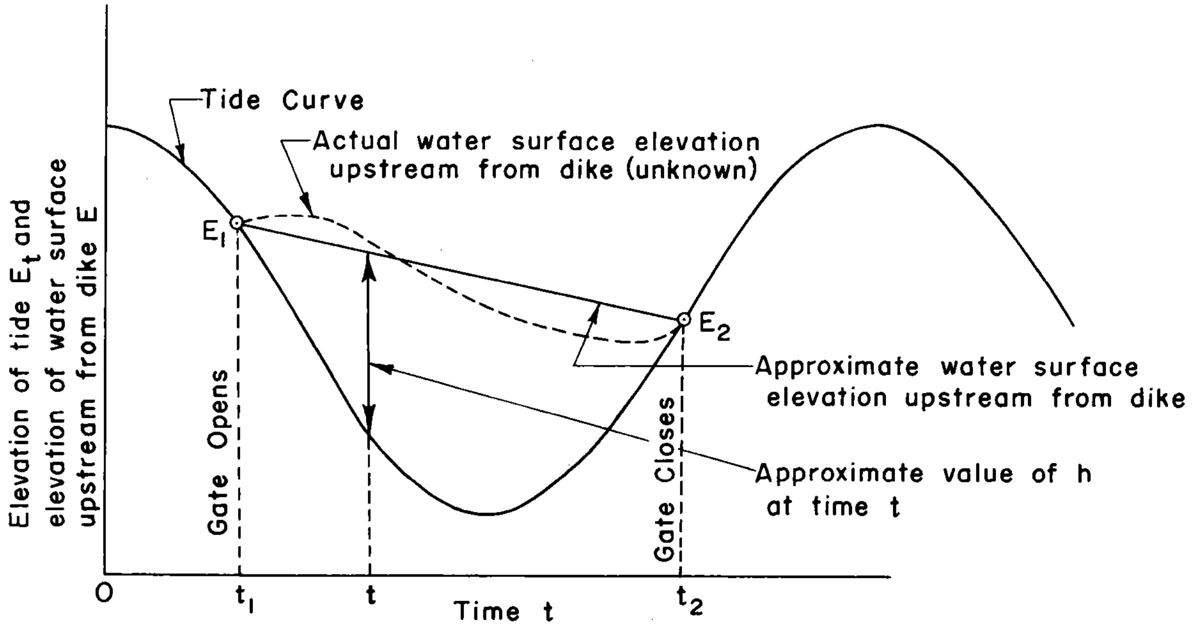

FIGURE 2

A straight line connecting points E_1 on the descending portion and E_2 on the ascending portion of the first cycle of the tide curve is an approximation of the water-surface elevation for the time interval $(t_2 - t_1)$ in which the outlet is discharging. The actual water-surface elevation is represented by the dashed curve shown in Fig. 2.

A close approximation of the rate of outflow Q through the pipe outlet is given by the relationship

$$Q = k A_p \sqrt{h} \tag{4}$$

where k is considered to be a constant.

The total volume of outflow $(O_2 - O_1)$ through the outlet is the integral of Q dt or

$$(O_2 - O_1) = \int_{t_1}^{t_2} Q \, dt = k A_p \int_{t_1}^{t_2} \sqrt{h} \, dt \tag{5}$$

Equating the total volume of outflow during the time interval $(t_2 - t_1)$ to the total volume of the inflow during one cycle of the tide i T results in the relationship

$$\frac{i\,T}{k\,A_p} = \int_{t_1}^{t_2} \sqrt{h}\;dt \equiv B \qquad (6)$$

where B represents the area under a curve that would be obtained in the plot of $\sqrt{h}$ vs t between the time t_1 and t_2. The values of h for the plot are obtained by measuring the ordinates between the approximate water-surface elevation (the straight line connecting E_1 and E_2) and the tide curve. Determination of the value of B is illustrated by Step 2b, ES-103, sheet 2.

Rewriting Eq. (6)

$$k\,A_p = \frac{i\,T}{B} \qquad (7)$$

The pipe size corresponding to the value of $k\,A_p$ in Eq. (7) can be ascertained from the rating curves of the pipe outlets. The values of $k\,A_p$ corresponding to various size pipes are obtained from the relationship (see Eq. 4).

$$k\,A_p = \frac{Q}{\sqrt{h}} \qquad (8)$$

A value of $k\,A_p$ is obtained when an arbitrarily selected value of h and the corresponding Q as read from the rating curves are substituted in Eq. (8).

The pipe size used to obtain a $k\,A_p$ value in Eq. (8) which is equal to the value of $k\,A_p$ in Eq. (7) is the approximate pipe size corresponding to the value of E_1. This method of determining the corresponding approximate pipe size to the values of E_1 and E_2 is illustrated by Steps 2d and 2e of ES-103, sheet 2.

Method of routing to determine the value of E_2 given E_1 and A_p. The storage equation is

$$\Delta I = \Delta O + \Delta S \qquad (9)$$

where ΔI = the volume of inflow in any fixed incremental time interval Δt
ΔO = the volume of outflow in any fixed incremental time interval Δt
ΔS = the volume of increased storage in any fixed incremental time interval Δt

The inflow rate i is constant and the volume of inflow for the time interval is $i\,\Delta t$. When Δt is sufficiently small, the volume of outflow ΔO for the time interval Δt is nearly

$$\Delta O = \frac{Q_a + Q_b}{2}\,\Delta t \qquad (10)$$

where Q_a = rate of outflow at the beginning of the time interval Δt
Q_b = rate of outflow at the end of the time interval Δt

Writing the storage equation in terms of inflow rate i and outflow rate Q for the incremental time interval Δt obtain, after dividing by Δt,

$$i = \frac{Q_a + Q_b}{2} + \frac{\Delta S}{\Delta t} \tag{11}$$

Solving for the rate of change of storage with respect to time, obtain

$$\frac{\Delta S}{\Delta t} = i - \frac{Q_a + Q_b}{2} \tag{12}$$

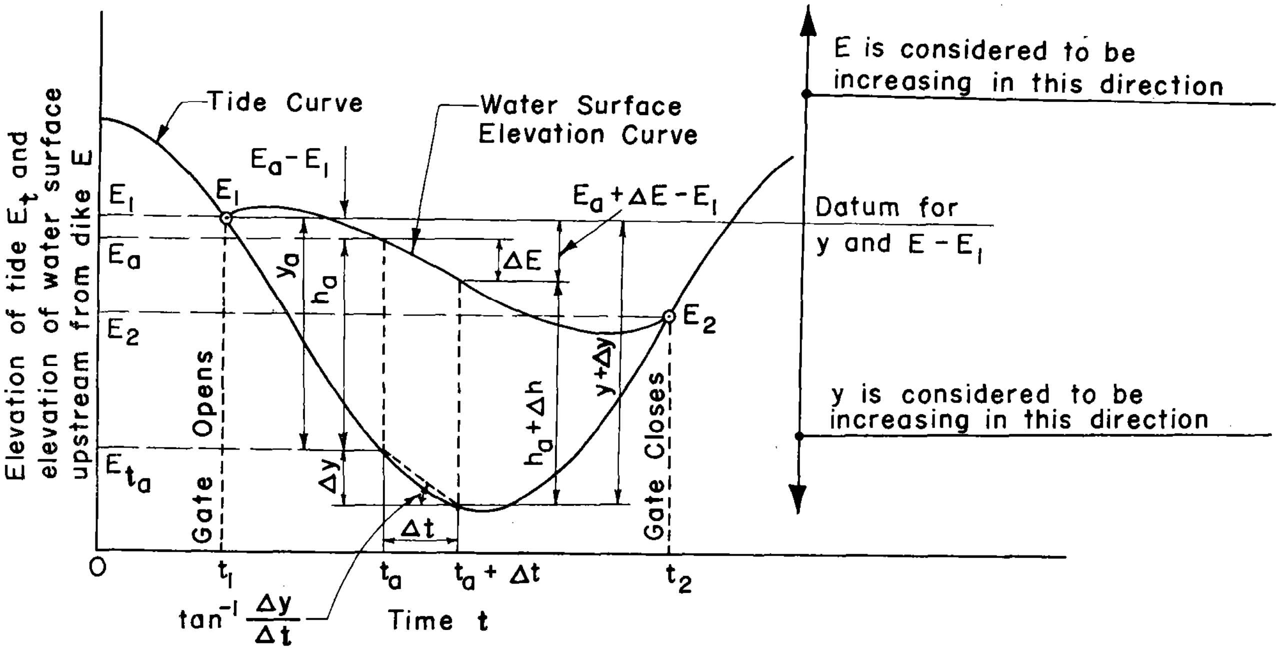

FIGURE 3

The head h_a causing discharge through the pipe (see Fig. 3) at time t_a is

$$h_a = E_a - E_{t_a} \equiv (E_a - E_1) + (E_1 - E_{t_a}) \tag{13}$$

Let $y = E_1 - E_t$. At the time $t_a + \Delta t$ the head $h_a + \Delta h$ is

$$h_a + \Delta h = \left[(E_a + \Delta E) - E_1\right] + \left[E_1 - (E_{t_a} + \Delta E_t)\right] \tag{14}$$

Let $\Delta y = -\Delta E_t$. Subtracting Eq. (13) from Eq. (14) and replacing $-\Delta E_t$ by Δy, write the result

$$\Delta h = \Delta y + \Delta E$$

Dividing by Δt

$$\frac{\Delta h}{\Delta t} = \frac{\Delta y}{\Delta t} + \frac{\Delta E}{\Delta t} \tag{15}$$

Solving for Δt

Solving for Δt

$$\Delta t = \frac{\Delta h}{\frac{\Delta y}{\Delta t} + \frac{\Delta E}{\Delta t}}$$

and since $\frac{\Delta E}{\Delta t} \equiv \frac{\frac{\Delta S}{\Delta t}}{\frac{\Delta S}{\Delta E}}$

$$\Delta t = \frac{\Delta h}{\frac{\Delta y}{\Delta t} + \frac{\frac{\Delta S}{\Delta t}}{\frac{\Delta S}{\Delta E}}} \tag{16}$$

Equation (16) is the basic equation used in routing through the tide gate. The value of $\frac{\Delta S}{\Delta E}$ is obtained from the storage curve. It is the rate of change of volume of storage at the elevation E_a. The value of $\frac{\Delta S}{\Delta t}$ is the rate of change of storage with respect to time during the time interval Δt and is evaluated by Eq. (12). The value of $\frac{\Delta y}{\Delta t}$ is the rate of change of the tide elevation as obtained from the tide curve. The value of Δh is arbitrarily chosen and the corresponding value of Δt is determined by Eq. (16). The method of routing is illustrated by Step 4 of ES-103, sheet 3.

EXAMPLE

Given: 1. An area of land subject to flooding by high tide.

2. The design inflow discharge from the land side is constant, $i = 50$ cfs.

3. The storage-elevation curve (known as the storage curve) of the site is given in Fig. 1. This curve is prepared from data obtained by a topographic survey of the area upstream from the proposed dike used to withhold the tidewater. The storage curve is plotted in units of cfs-min to eliminate conversion factors in routing the inflow discharge i. The proposed dike to withhold tidewater will be called the dike throughout the rest of this example.

4. The design tide vs time curve (known as the tide curve) is given in Fig. 2. It is the design relationship of the tide vs time and is cyclic. The time zero ($t = 0$) is taken at high tide.

5. Figure 3 gives rating curves for various sizes of corrugated metal pipes with automatic drainage gates. The rating curves are for the length of pipe needed at this site.

6. The corrugated metal pipe with the automatic drainage gate is to be placed below low tide water elevation. The corrugated metal pipe used to discharge the inflow $i = 50$ cfs and the associated automatic drainage gate will be called the pipe and the gate throughout the rest of this example.

7. Benefit vs elevation curves are shown in Fig. 4. They are the results of data obtained by the topographic survey and estimated value of the reclaimed land.

8. The estimated costs of pipes of various sizes in place are given in Table 1 and the estimated initial fixed cost is $1500.

Determine: 1. The elevations E_2 at which the gate closes corresponding to the elevations E_1 at which the gate opens. Take E_1 = 23.75; 24.00; 24.50; 25.00; 25.50; 26.00; 26.50; 27.00; and 27.50.

2. The approximate pipe size corresponding to any elevation E_1 of the water surface at which the gate opens.

3. Pipe size or number and size of pipes in a battery that will give greater approximated benefits than costs.

4. The water-surface elevation E upstream from the dike vs time t for a 48" pipe.

5. The highest elevation of flooding E_m for the size 42" and 48" pipe.

Solution: 1. Solve for the elevations of the water surface at the time the automatic drainage gate closes E_2 corresponding to the elevations of the water surface at the time the gate opens E_1.

It is known that the amount of inflow from the time t_2 at which the gate closes until the time t_3 at which it opens again is

$$I_3 - I_2 = i\,(t_3 - t_2)$$

Since no outflow occurred in this interval the storage has increased by the amount of inflow or

$$S_3 - S_2 = i\,(t_3 - t_2)$$

When the elevation of the water surface $E_1 = E_3$ at which the gate is to open is selected, the elevation of the water surface E_2 at which the gate must have closed can be determined because the storage upstream from the dike is known at every elevation.

a. Prepare on tracing paper a curve giving the relationship of water-surface elevation vs time when the inflow to the dike is $i = 50$ cfs and the outflow is zero. The amount of storage is arbitrarily taken to be zero at time zero. The water-surface elevations are to be plotted as ordinates and to the same scale as the tide curve (Fig. 2). Time is plotted along the abscissa to the same scale as the time scale of the tide curve. The plot is shown in Fig. 5. The time is determined by the relation $t = S/i$. Values of S corresponding to any elevation E of the water surface upstream from the dike is read from Fig. 1.

b. Superimpose the E vs t curve prepared in the preceding step over the tide curve (Fig. 2) keeping the ordinates or values of water-surface-elevation scales matched. Move the superimposed curve horizontally until it intersects the descending portion of the second cycle of the tide curve at a selected value of E_1. Read the value of E_2 at the point the curve intersects the ascending portion of the first cycle of the tide curve. Note that the minimum value of E_2 is at the point the tide curve and the superimposed curve become tangent. Various selected values of E_1 and the corresponding values of E_2 are

E_1	23.75	24.00	24.50	25.00	25.50	26.00	26.50	27.00	27.50
E_2	17.40	19.63	21.64	22.85	23.78	24.64	25.42	26.18	26.91

2. Solve for the approximate pipe size corresponding to each selected value of E_1.

The total volume of inflow in one cycle of tide which is equal to $i\,T$ must be the volume of outflow discharge O in the time interval $(t_2 - t_1)$ in which the gate is open during that cycle.

$$O = i\,T$$

where T = time interval of one cycle of the design tide curve in mins.

The outflow rate through the pipe and gate is given by the approximate relationship

$$Q = k\,A_p\sqrt{h}$$

where
- Q = discharge rate through the pipe in cfs
- k = a constant
- h = $E - E_t$ in ft
- E = elevation of the water surface on the upstream side of the dike in ft
- E_t = elevation of the tide in ft
- A_p = cross-sectional area of pipe in ft^2

The total volume of outflow O through the pipe is

$$O = \int_{t_1}^{t_2} Q\,dt = k\,A_p\int_{t_1}^{t_2}\sqrt{h}\,dt$$

But $O = i\,T$ or

$$\frac{i\,T}{k\,A_p} = \int_{t_1}^{t_2}\sqrt{h}\,dt$$

REFERENCE

a. Locate the selected values of E_1 on the descending portion of the first cycle and corresponding values of E_2 on the ascending portion of the first cycle of the tide curve (Fig. 2). Draw lines connecting corresponding values of E_1 and E_2. The value of E_1 is the elevation of the water surface at which the gate opens and E_2 the elevation of the water surface at which the gate closes. Each of the lines connecting pairs of E_1 and E_2 is an approximation of the water-surface-elevation curve if the pipe is the proper size.

b. Solve for the value of

$$\int_{t_1}^{t_2} \sqrt{h}\ dt \equiv B$$

For each selected value of E_1 find the approximate head h on the discharge pipe at equal time intervals Δt. This head h is found by scaling the distance between the straight line representing the approximate elevation of the water surface and the tide curve representing the elevation of the tide. Tabulate values of the approximate h and prepare Table 2.

(i) The value of h is tabulated in Col. 1 of Table 2. To simplify these measurements the first interval is not always taken to be Δt. For example when $E_1 = 23.75$ the first interval is taken to be 16 mins. instead of $\Delta t = 20$ mins. because each subsequent value of h can then be measured along an ordinate which is a line. The head h is measured in ft.

(ii) Column 2 lists the square root of Col. 1.

(iii) Column 3 lists a correction that is to be added to the summation of Col. 2. It contains the end corrections for the beginning and ending intervals Δt_b that are shorter than Δt. The end correction is equal to $\frac{\sqrt{h}}{2}\left[\frac{\Delta t_b - \Delta t}{\Delta t}\right]$.

Example of correction: When $E_1 = 23.75$ ft the time interval between $h = 0$ and $h = 0.65$ is 16 mins. The correction is $\frac{1}{2}\left[\frac{16 - \Delta t}{\Delta t}\right]\sqrt{0.65} \equiv -0.081$. The time interval between $h = 0.6$ and $h = 0$ is 10 mins. $\frac{1}{2}\left[\frac{10 - \Delta t}{\Delta t}\right]\sqrt{0.6} \equiv -0.194$.

The solution in Table 2 is a method of integrating by steps the area under a curve that would be obtained in the plot of $\sqrt{h}$ as ordinates vs t as abscissas between the time t_1 and t_2.

(iv) Solve for the value of

$$\sum_{t_1}^{t_2} \sqrt{h}\ \Delta t = \Delta t \sum_{t_1}^{t_2} \sqrt{h} = \left[(\text{sum of Col. 2}) + (\text{sum of Col. 3})\right] \Delta t \equiv B$$

where h = value of head at the end of the time interval Δt.
$t_2 - t_1$ = total time gate is open during one cycle of tide.
t_1 = time the gate opens.
t_2 = time the gate closes.

c. Solve for the values of $\frac{i\,T}{B} = k\,A_p$ for each selected value of E_1 where $i = 50$ cfs and B = value determined in Step 2b(iv).

(i) Column 1 lists values of E_1.

(ii) Column 2 lists values of B.

(iii) Column 3 lists values of $\frac{i\,T}{B} = k\,A_p$.

$i\,T = 50\ (720) = 36{,}000$ cfs-mins.

d. Prepare a set of curves (Fig. 6) giving the relationship of $k\,A_p$ vs A_p for various sizes of pipes and the size of pipes in batteries. The rating curves follow closely the relationship

$$Q = k\,A_p\sqrt{h}$$

where Q = discharge of pipe of required length for the site and with a gate attached in cfs.
A_p = cross-sectional area of pipe in ft^2.
k = a constant.
h = operating head of the pipe in ft.

The value of $k\,A_p$ obtained from the rating curve is $\frac{Q}{\sqrt{h}}$ where h is arbitrarily selected. The value of h arbitrarily selected can be obtained by inspection of Fig. 2 and should be about the average head over the pipe. In this example h was assumed to be 9 ft. Coordinates for the curve associated with one pipe are values of $k\,A_p$ and A_p. One point is plotted on this curve from each rating curve. The coordinates of the curve associated with m pipes of the same sizes are $m\,(k\,A_p)$ and $m\,A_p$ where $k\,A_p$ and A_p are the coordinates of the curve associated with one pipe.

e. Solve for the value of A_p and prepare Fig. 7.

(i) For each value of E_1 enter Fig. 6 with the value of $k\,A_p$ as determined in Step 2 and read the corresponding value of A_p according to the number of pipes in the battery.

(ii) Plot the value of E_1 as ordinates and the values of A_p of various numbers of pipes as abscissas. The value of A_p is the total cross-sectional area of all the pipes in a battery.

(iii) Prepare line scales of pipe sizes vs cross-sectional areas for the number of pipes in a battery. In Fig. 7 six lines having 1, 2, 3, 4, 5, and 6 pipes are given. One side of each scale line gives the values of A_p and the other side of the line gives the corresponding pipe diameter. The pipe size and number of pipes required to discharge the inflow i when the gate opens at any elevation E_1 may be read from Fig. 7.

3. Solve for the pipe size or number and size of pipes in a battery that will give greater approximate benefits than cost. The words "approximate benefits" are used here in the sense that the maximum elevation of flooding is taken to be E_1. As will be seen later the maximum elevation of flooding occurs at a slightly higher elevation than E_1. (See Step 5.) The value of A_p as determined by the approximate method in Step 2 is checked for accuracy in Step 4.

a. Prepare Table 4.

(i) Column 1 lists number of pipes in batteries as read from Fig. 7.

(ii) Column 2 lists size of pipes in battery corresponding to number of pipes.

(iii) Column 3 lists the elevation E_1 corresponding to the area of pipes in Col. 2. Read from Fig. 7.

(iv) Column 4 lists acres benefited as obtained from Fig. 4 at values of E_1 tabulated in Col. 3.

(v) Column 5 is read from Fig. 4 at values tabulated in Col. 3.

(vi) Column 6 is read from Table 1.

(vii) Column 7 is given. (Item 8 in Given.)

(viii) Column 8 is the sum of Col. 6 and Col. 7.

(ix) Column 9 = Col. 5 − Col. 8.

b. The results of Table 4 show that one 42" pipe and one 48" pipe have approximate benefits greater than costs.

REFERENCE

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ENGINEERING DIVISION - DESIGN SECTION

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4. Draw the water-surface elevation vs time curve and check the accuracy of the relationship of E_1 vs A_p for a 48" pipe as approximated in Step 2, Fig. 7.

a. Prepare tabular form given by Table 5. Each set of horizontal lines in this table should be completed before starting the following set of lines. The value of E_1 for 48" pipe is 23.95 ft. (See Fig. 7.)

(i) Column 1 lists arbitrarily selected values of the head h on the outlet pipe.

(ii) Column 2 lists the rate of outflow Q for the selected value of h in Col. 1. This value is read from the rating curve (Fig. 3) for the size and length of pipe under consideration.

(iii) Column 3 is the summation of two consecutive values in Col. 2.

(iv) Column 4 is Col. 3 divided by 2. This is the average rate of outflow Q between two consecutive heads listed in Col. 1.

(v) Column 5 is obtained by subtracting the values in Col. 4 from the inflow rate, i = 50 cfs. This column will have positive values when the outflow rate is less than the inflow rate. This occurs when the value of h is small.

(vi) Column 6 lists the slope $\frac{\Delta S}{\Delta E}$ of the storage curve (Fig. 1). The value of $\frac{\Delta S}{\Delta E}$ varies at every elevation. An average value of the slope $\frac{\Delta S}{\Delta E}$ is taken for a small change in storage and its value is assumed to be constant for this range. The range of elevation for which the rate $\frac{\Delta S}{\Delta E}$ is applicable is tabulated in parenthesis in Col. 6. This tabulation of range of elevation enables a check to be made to determine the validity of this value of $\frac{\Delta S}{\Delta E}$ as calculations are advanced. The value of $\frac{\Delta S}{\Delta E}$ is always positive.

(vii) Column 7 lists the slope $\frac{\Delta y}{\Delta t}$ of the tide curve at the time under consideration.

$$y = E_1 - E_t$$

where E_t = elevation of the tide.
E_1 = water elevation upstream from the dike when the gate opens.

The value of $\frac{\Delta y}{\Delta t}$ is the negative of the slope of the tide curve. When the tide is decreasing in elevation $\frac{\Delta y}{\Delta t}$ is positive and when the tide is rising $\frac{\Delta y}{\Delta t}$ is negative. The proper algebraic sign is essential in Col. 7. The corresponding tide elevations used in evaluating $\frac{\Delta y}{\Delta t}$ are tabulated in parenthesis in Col. 7.

(viii) Column 8 lists the interval Δh between consecutive values of h of Col. 1. The proper sign must be prefixed to this value. These signs will be positive as the values of h increase in Col. 1 and negative when the values of h decrease.

(ix) Column 9 is Col. 5 divided by Col. 6. The sign of values in this column is the same sign found in Col. 5.

(x) Column 10 is the sum of Col. 7 and Col. 9. The sign of this column is determined by the signs and values of Cols. 7 and 9.

(xi) Column 11 is Col. 8 divided by Col. 10. The sign will always be positive.

(xii) Column 12 is the summation of Δt. The time corresponding to E_1 on the descending portion of the tide curve (Fig. 2) should be listed at the beginning of Col. 12.

(xiii) Column 13 is the product of Col. 5 and Col. 11.

(xiv) Column 14 is the volume of stored water upstream from the dike. The storage (Fig. 1) corresponding to E_1 on the descending portion of the tide curve is listed at the beginning of Col. 14. The water stored at elevation 23.95 ft is 21,400 cfs-min. Column 13 is used to obtain the accumulated storage in Col. 14.

(xv) Column 15 is the product of Col. 7 and Col. 11.

(xvi) Column 16 lists the elevation E_t of the tide curve as determined by Col. 15. The value of E_1 is listed at the beginning of this column. Note that Δy is considered positive when the elevation of the tide curve E_t is decreasing, therefore when the value of Δy in Col. 15 is positive, it should be subtracted from the previous E_t; when negative it should be added to E_t. The values in this column and Col. 12 are used as coordinates to plot a point on the tide graph. It is at this point in the computations that the value of h and $\frac{\Delta y}{\Delta t}$ for the next time interval are to be selected. If the plotted points of Cols. 12 and 16 do not fall on the tide curve, and if the actual slope of the tide curve were used for $\frac{\Delta y}{\Delta t}$ for the remaining intervals, the curve of plotted points would parallel the tide curve instead of approaching it. The plotted points can be made to approximate the tide curve by the following procedure. The next value of h is selected by projecting the water-surface curve for a time interval Δt and measuring vertically between the projected water-surface curve and a point on the tide curve which is called point a in the next two sentences. The value of $\frac{\Delta y}{\Delta t}$ is the slope of the line connecting the plotted point having coordinates as determined from Col. 12 and Col. 16 to the point a on the tide curve (see Fig. 8). When a point having coordinates of Cols. 12 and 16 plots near the minimum elevation of the tide curve, a slope $\frac{\Delta y}{\Delta t}$ of zero can be used by selecting h such that the point a falls on the ascending portion of the tide curve.

(xvii) Column 17 is the elevation E of the water surface upstream from the dike. It is obtained from the storage curve (Fig. 1). These values correspond to the values S given in Col. 14. The values of Cols. 14 and 12 are used as coordinates to plot the water surface upstream from the dike in Fig. 8.

(xviii) Column 18 lists a check value for h. If Col. 18 varies significantly from Col. 1, a mistake has probably been made in the computations. Values of Col. 18 are obtained by subtracting Col. 16 from Col. 17.

$$h = E - E_t$$

After completing Table 5, the value of E_2 corresponding to the value of E_1 used at the beginning of Col. 16 should be determined by the procedure of Step 1. If the computed value of E_2 obtained by Step 1 is not approximately the same as the routed value obtained by Step 4, then the value of E_1 corresponding to A_p is incorrectly approximated by the procedure of Step 2. When the routed value of E_2 is significantly different from the computed value of E_2, the value of E_1 corresponding to A_p should be recalculated (Step 1) to correspond to the routed value of E_2. Step 4 should be repeated using the new value of E_1.

b. The computations for one 48" pipe are given in Table 5 and the water-surface curve has been plotted in Fig. 8. The curve for one 42" pipe is also plotted in Fig. 8, but the computations are not shown.

5. a. When the head on the outlet pipe is small, the inflow exceeds the outflow and causes a rise in the water-surface curve (Fig. 8). Only land lying above the maximum water surface E_m would be free of flooding for the given conditions. This elevation for the 42" CMP is 25.21 ft and for the 48" CMP the elevation is 24.02 ft.

b. Following is a comparison of the values of E_1 and E_2 for the approximate solution and the routing method.

Pipe Size Diameter--inches	Selected E_1	Value of E_2 by computing from E_1 (Step 1)	Value of E_2 by routing from E_1 (Step 4)	Value of E_m read from Fig. 8
42"	25.10	23.04	23.10	25.21
48"	23.95	19.30	19.32	24.02

REFERENCE

HYDRAULICS: ROUTING THROUGH TIDE GATES; Example

FIGURE 1

STORAGE CURVE OF SITE

Relationship of elevation vs storage capacity upstream from dike

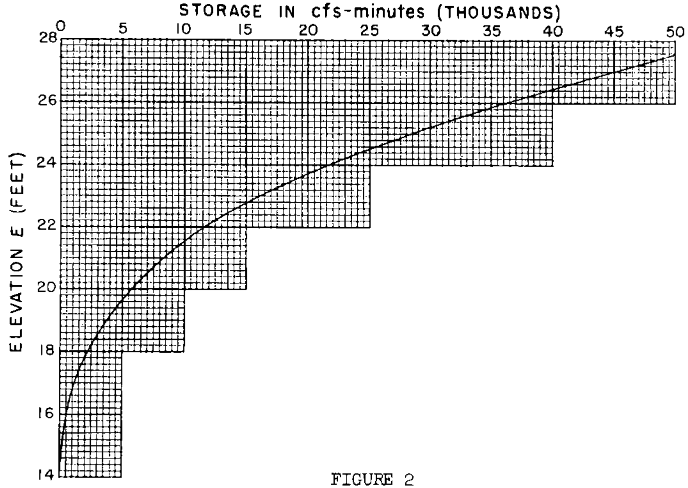


FIGURE 3

RATING CURVES FOR A PARTICULAR LENGTH OF SUBMERGED CMP WITH AUTOMATIC DRAINAGE GATES

Relationship of outflow discharge vs head on the pipe

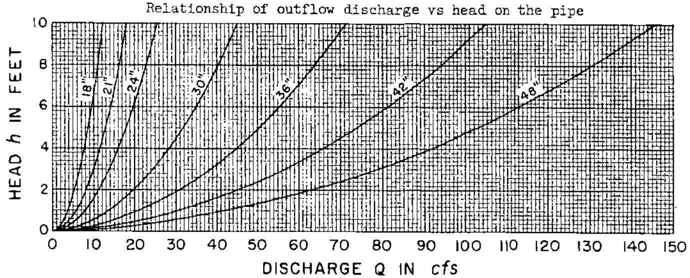


FIGURE 2

DESIGN TIDE CURVE

Relationship of elevation of tide at dike vs time

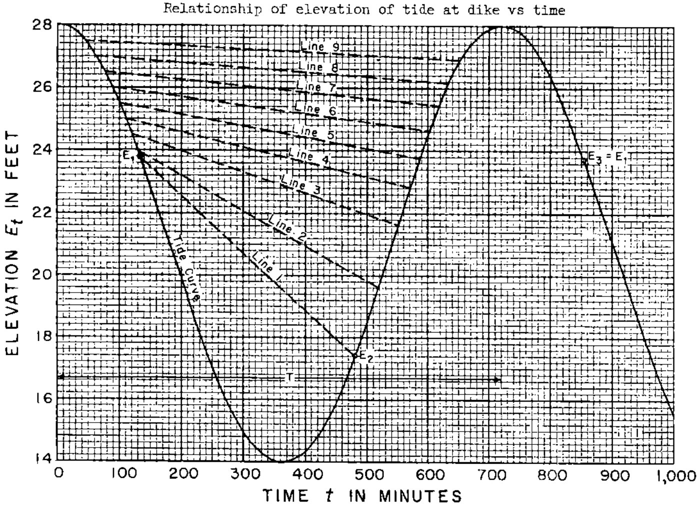
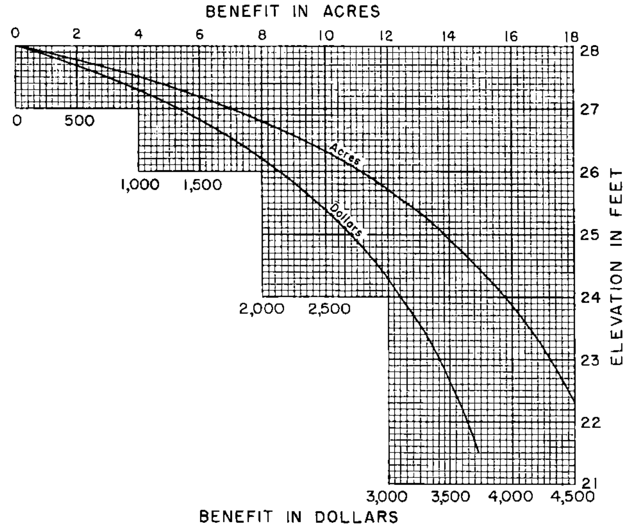


FIGURE 4

RELATIONSHIP OF ELEVATION VS BENEFIT



REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES-103

SHEET 4 OF 7

DATE 8-10-55

FIGURE 5

ELEVATION VS STORAGE CURVE EXPRESSED AS
ELEVATION VS TIME WHEN OUTFLOW IS ZERO

(Datum of time in this plot is not the same as given in the tide curve.) This curve should be prepared on transparent graph paper. Superimposing this curve on the tide curve (Fig. 2) gives the value of E_2 corresponding to $E_3 = E_1$ when the ordinates are kept equal.

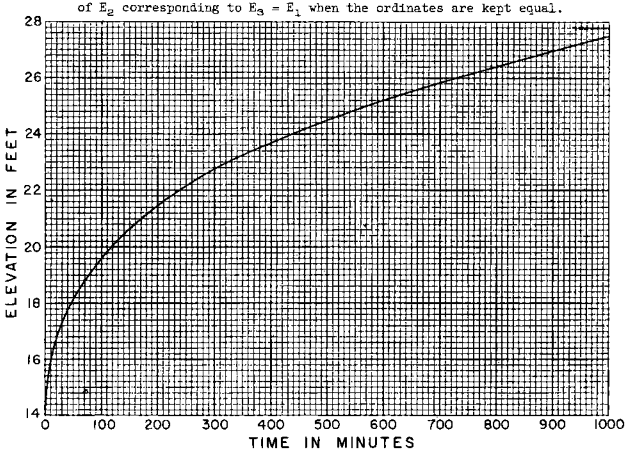


FIGURE 7

RELATIONSHIP OF E_1 VS A_p AND RELATIONSHIP OF VALUE OF A_p IN A BATTERY OF PIPES OF EQUAL SIZES

E_1 vs A_p prepared from Fig. 6 and Table 3

A_p vs pipe size prepared by calculating cross-sectional areas

Values tabulated on the top side of scale lines are the size of the pipe. Graduations on the lower side of scale lines give the area of a battery of pipes. Read values at the bottom of the figure. Each scale line refers to a given number of pipes of equal size in the battery.

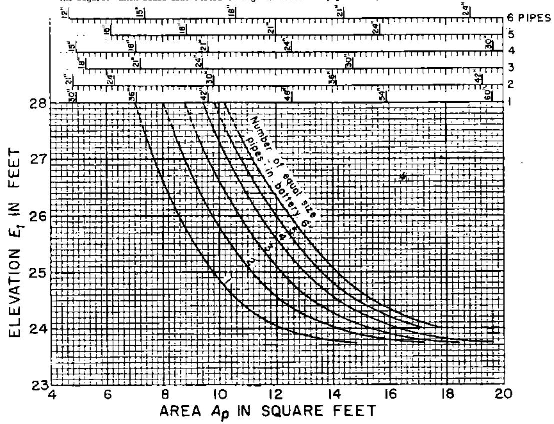


FIGURE 6

RELATIONSHIP OF $\frac{Q}{\sqrt{h}} = k\,A_p$ VS A_p FOR BATTERIES OF VARIOUS SIZE CMP

Obtained from rating curve (Fig. 3)

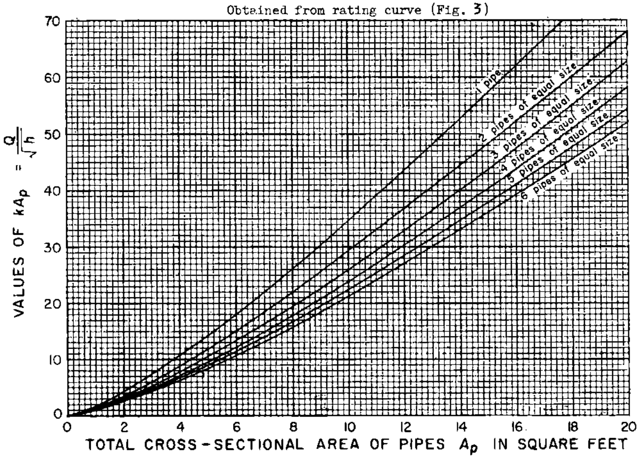


FIGURE 8

WATER SURFACE UPSTREAM FROM DIKE VS TIME OBTAINED BY ROUTING

See Step 4 and Table 5

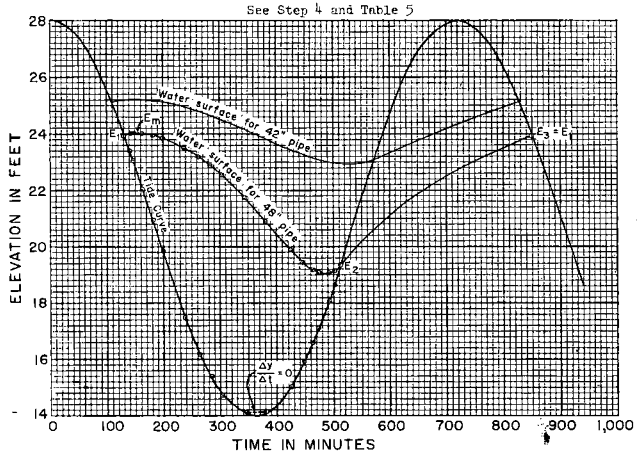


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TABLE 2

Approximate Values of $B \equiv \int_{t_1}^{t_2} \sqrt{h}\, dt$ Corresponding to Selected Values of E_1

Line 1 — $E_1 = 23.75$, $E_2 = 17.40$, $\Delta t = 20$ min.

1 h	2 $\sqrt{h}$	3 End Correction
0	0	(16)
0.65	0.806	− 0.081
1.50	1.225	
2.30	1.517	
3.20	1.789	
4.00	2.000	
4.60	2.145	
5.30	2.302	
5.70	2.387	
5.90	2.429	
5.90	2.429	
5.70	2.387	
5.30	2.302	
4.80	2.191	
4.10	2.025	
3.10	1.761	
2.00	1.414	
0.60	0.775	− 0.194
0	0	(10)
	31.884	− 0.275

31.884 − 0.275 = 31.609

B = 31.609 (20) = 632.18

Line 2 — $E_1 = 24.00$, $E_2 = 19.63$, $\Delta t = 20$ min.

1 h	2 $\sqrt{h}$	3 End Correction
0	0	0
0.95	0.975	
1.90	1.378	
2.85	1.688	
3.90	1.975	
4.85	2.202	
5.65	2.377	
6.40	2.530	
6.98	2.642	
7.35	2.711	
7.50	2.739	
7.50	2.739	
7.30	2.702	
6.90	2.627	
6.30	2.510	
5.50	2.345	
4.50	2.121	
3.30	1.817	
2.00	1.414	
0.60	0.775	− 0.233
0	0	(8)
	40.267	− 0.233

40.267 − 0.233 = 40.034

B = 40.034 (20) = 800.68

Line 3 — $E_1 = 24.50$, $E_2 = 21.64$, $\Delta t = 20$ min.

1 h	2 $\sqrt{h}$	3 End Correction
0	0	0
0.98	0.990	
2.01	1.418	
3.05	1.746	
4.12	2.030	
5.25	2.291	
6.25	2.500	
7.11	2.666	
7.81	2.795	
8.42	2.902	
8.78	2.963	
8.98	2.997	
8.95	2.992	
8.70	2.950	
8.30	2.881	
7.70	2.775	
6.85	2.617	
5.85	2.419	
4.75	2.179	
3.41	1.847	
2.10	1.449	
0.68	0.825	− 0.206
0	0	(10)
	48.232	− 0.206

48.232 − 0.206 = 48.026

B = 48.026 (20) = 960.52

Line 4 — $E_1 = 25.00$, $E_2 = 22.85$, $\Delta t = 20$ min.

1 h	2 $\sqrt{h}$	3 End Correction
0	0	0
0.92	0.959	
2.02	1.421	
3.10	1.761	
4.20	2.049	
5.35	2.313	
6.45	2.540	
7.41	2.722	
8.25	2.872	
8.95	2.992	
9.42	3.069	
9.71	3.116	
9.89	3.145	
9.80	3.130	
9.50	3.082	
9.02	3.003	
8.40	2.898	
7.50	2.739	
6.41	2.532	
5.30	2.302	
3.95	1.987	
2.65	1.629	
1.35	1.162	
0	0	0
	53.423	0

53.423 − 0 = 53.423

B = 53.423 (20) = 1068.46

Line 5 — $E_1 = 25.50$, $E_2 = 23.78$, $\Delta t = 20$ min.

1 h	2 $\sqrt{h}$	3 End Correction
0	0	0
0.91	0.954	
1.91	1.382	
3.08	1.755	
4.12	2.030	
5.28	2.298	
6.48	2.546	
7.55	2.748	
8.48	2.912	
9.22	3.036	
9.90	3.146	
10.30	3.209	
10.51	3.242	
10.59	3.254	
10.40	3.225	
10.08	3.175	
9.55	3.090	
8.75	2.958	
7.85	2.802	
6.78	2.604	
5.51	2.347	
4.22	2.054	
2.92	1.709	
1.70	1.304	
0.35	0.592	− 0.207
0	0	(6)
	58.372	− 0.207

58.372 − 0.207 = 58.165

B = 58.165 (20) = 1163.30

Line 6 — $E_1 = 26.00$, $E_2 = 24.64$, $\Delta t = 20$ min.

1 h	2 $\sqrt{h}$	3 End Correction
0	0	0
0.82	0.906	
1.90	1.378	
3.02	1.738	
4.16	2.040	
5.28	2.298	
6.50	2.550	
7.60	2.757	
8.60	2.933	
9.50	3.082	
10.22	3.197	
10.75	3.279	
11.08	3.329	
11.28	3.359	
11.21	3.348	
10.98	3.314	
10.52	3.243	
9.90	3.146	
9.10	3.017	
8.05	2.837	
6.95	2.636	
5.68	2:383	
4.40	2.098	
3.10	1.761	
1.90	1.378	
0.65	0.806	− 0.161
0	0	(12)
	62.813	− 0.161

62.813 − 0.161 = 62.652

B = 62.652 (20) = 1253.04

Line 7 — $E_1 = 26.50$, $E_2 = 25.42$, $\Delta t = 20$ min.

1 h	2 $\sqrt{h}$	3 End Correction
0	0	(3)
0.10	0.316	− 0.134
0.95	0.975	
1.90	1.378	
3.00	1.732	
4.15	2.037	
5.21	2.283	
6.41	2.532	
7.60	2.757	
8.70	2.950	
9.65	3.106	
10.48	3.237	
11.17	3.342	
11.59	3.402	
11.82	3.438	
11.91	3.451	
11.79	3.434	
11.48	3.388	
10.95	3.309	
10.21	3.195	
9.30	3.050	
8.28	2.877	
7.05	2.655	
5.80	2.408	
4.55	2.133	
3.30	1.817	
2.00	1.414	
0.91	0.954	− 0.048
0	0	(18)
	67.570	− 0.182

67.570 − 0.182 = 67.388

B = 67.388 (20) = 1347.76

Line 8 — $E_1 = 27.00$, $E_2 = 26.18$, $\Delta t = 20$ min.

1 h	2 $\sqrt{h}$	3 End Correction
0	0	(18)
0.60	0.775	− 0.039
1.50	1.225	
2.45	1.565	
3.50	1.871	
4.62	2.149	
5.80	2.408	
6.91	2.629	
8.15	2.855	
9.30	3.050	
10.25	3.202	
11.01	3.318	
11.79	3.434	
12.20	3.493	
12.48	3.533	
12.59	3.548	
12.45	3.528	
12.15	3.486	
11.65	3.413	
10.90	3.302	
10.05	3.170	
9.00	3.000	
7.80	2.793	
6.58	2.565	
5.28	2.298	
4.10	2.025	
2.80	1.673	
1.75	1.323	
0.62	0.787	− 0.138
0	0	(13)
	72.418	− 0.177

72.418 − 0.177 = 72.241

B = 72.241 (20) = 1444.82

Line 9 — $E_1 = 27.50$, $E_2 = 26.91$, $\Delta t = 20$ min.

1 h	2 $\sqrt{h}$	3 End Correction
0	0	(15)
0.35	0.592	− 0.074
1.10	1.049	
1.92	1.386	
2.90	1.703	
4.00	2.000	
5.15	2.269	
6.29	2.508	
7.45	2.729	
8.71	2.951	
9.85	3.138	
10.81	3.288	
11.62	3.409	
12.37	3.517	
12.80	3.578	
13.08	3.617	
13.20	3.633	
13.10	3.619	
12.80	3.578	
12.30	3.507	
11.60	3.406	
10.70	3.271	
9.70	3.114	
8.50	2.915	
7.25	2.693	
6.00	2.449	
4.80	2.191	
3.50	1.871	
2.40	1.549	
1.40	1.183	
0.45	0.671	− 0.134
0	0	(12)
	77.384	− 0.208

77.384 − 0.208 = 77.176

B = 77.176 (20) = 1543.52

TABLE 1

Cost Schedule for CMP
with Automatic Drainage Gate in Place on the Site

Diameter \ Number of Pipes	1	2	3	4	5	6
12"	300	600	900	1200	1500	1800
15"	360	720	1080	1440	1800	2160
18"	432	864	1296	1728	2160	2592
21"	489	978	1467	1956	2445	2934
24"	555	1110	1665	2220	2775	3330
30"	702	1404	2106	2808	3510	
36"	804	1608	2412	3216		
42"	986	1972	2958			
48"	1240	2480				

TABLE 3

Approximate Values of $k A_p$
Corresponding to Selected Values of E_1

$$k A_p = \frac{i\, T}{B}$$

1 E_1--ft	2 B	3 $k A_p$
23.75	632.18	56.95
24.00	800.68	44.96
24.50	960.52	37.48
25.00	1068.46	33.69
25.50	1163.30	30.95
26.00	1253.04	28.73
26.50	1347.76	26.71
27.00	1444.82	24.92
27.50	1543.52	23.32

TABLE 4

Cost of Batteries of Equal Size Pipe and Approximate Benefits

1 Number of Pipes	2 Pipe Size in.	3 E_1 ft	4 Benefit in Acres	5 Benefit in Dollars	6 Pipe Cost in Place Dollars	7 Fixed Initial Cost Dollars	8 Total Cost Dollars	9 Benefit – Cost Dollars
4	21	27.77	2.01	370	1956	1500	3456	– 3086
6	18	27.51	4.00	740	2592	1500	4092	– 3352
3	24	27.17	6.15	1160	1665	1500	3165	– 2005
5	21	25.97	11.23	2160	2445	1500	3945	– 1785
2	30	25.93	11.35	2180	1404	1500	2904	– 724
4	24	25.22	13.30	2580	2220	1500	3720	– 1140
1	42	25.10	13.60	2645	986	1500	2486	+ 159
6	21	24.85	14.11	2775	2934	1500	4434	– 1659
5	24	24.27	15.29	3010	2775	1500	4275	– 1265
3	30	24.10	15.60	3070	2106	1500	3606	– 536
2	36	24.00	15.76	3108	1608	1500	3108	000
1	48	23.95	15.90	3125	1240	1500	2740	+ 385

TABLE 5

Routing for a 48" Corrugated Metal Pipe.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
h ft	Q cfs	$Q_a + Q_b$ cfs	$\frac{Q_a + Q_b}{2} = \bar{Q}$ cfs	$\frac{\Delta S}{\Delta t} = i - \bar{Q}$ cfs	$\frac{\Delta S}{\Delta E}$	$\frac{\Delta y}{\Delta t}$	Δh ft	$\frac{\Delta S / \Delta t}{\Delta S / \Delta E}$	$\frac{\Delta y}{\Delta t} + \frac{\Delta S / \Delta t}{\Delta S / \Delta E}$	Δt min.	$\Sigma \Delta t$ min.	ΔS cfs-min.	$\Sigma \Delta S$ cfs-min.	Δy ft	E_t ft	E ft	h (Check) ft
			$\frac{3}{2}$	1 − 4				$\frac{5}{6}$	7 + 9	$\frac{8}{10}$		5 x 11		7 x 11		Elev. for 14	17 − 16
0	0										131.00		21,400		23.95	23.95	0
		30.0	15.0	+ 35.0	(24.28 23.60) + 6029	(23.95 22.00) + 0.0591	0.6	+ 0.0058	+ 0.0649	9.24		+ 323		+ 0.55			
0.6	30.0										140.24		21,723		23.40	24.00	0.60
		71.0	35.5	+ 14.5	+ 6029	+ 0.0591	0.4	+ 0.0024	+ 0.0615	6.50		+ 94		+ 0.38			
1.0	41.0										146.74		21,817		23.02	24.02	1.00
		103.4	51.7	− 1.7	+ 6029	+ 0.0591	1.0	− 0.0003	+ 0.0588	17.01		− 29		+ 1.01			
2.0	62.4										163.75		21,788		22.01	24.01	2.00
		140.6	70.3	− 20.3	+ 6029	(22.00 17.50) + 0.0607	1.0	− 0.0034	+ 0.0573	17.45		− 354		+ 1.06			
3.0	78.2										181.20		21,434		20.95	23.95	3.00
		169.4	84.7	− 34.7	+ 6029	+ 0.0607	1.0	− 0.0058	+ 0.0549	18.21		− 632		+ 1.11			
4.0	91.2										199.41		20,802		19.84	23.84	4.00
		203.4	101.7	− 51.7	(23.60 23.40) + 5830	+ 0.0607	2.0	− 0.0089	+ 0.0518	38.61		− 1996		+ 2.34			
6.0	112.2										238.02		18,806		17.50	23.50	6.00
		233.8	116.9	− 66.9	(23.40 22.80) + 5380	(17.50 16.20) + 0.0522	1.0	− 0.0124	+ 0.0398	25.13		− 1681		+ 1.31			
7.0	121.6										263.15		17,125		16.19	23.19	7.00
		247.6	123.8	− 73.8	+ 5380	(16.19 15.00) + 0.0370	0.5	− 0.0137	+ 0.0233	21.46		− 1584		+ 0.79			
7.5	126.0										284.61		15,541		15.40	22.90	7.50
		254.6	127.3	− 77.3	(22.88 22.40) + 4580	(15.40 14.70) + 0.0319	0.3	− 0.0169	+ 0.0150	20.00		− 1546		+ 0.64			
7.8	128.6										304.61		13,995		14.76	22.56	7.80
		255.6	127.8	− 77.8	(22.40 21.50) + 4020	(14.70 14.10) + 0.0146	− 0.2	− 0.0194	− 0.0048	41.67		− 3240		+ 0.61			
7.6	127.0										346.28		10,755		14.15	21.75	7.60
		247.0	123.5	− 73.5	(21.72 20.70) + 3195	0	− 0.8	− 0.0230	− 0.0230	34.78		− 2556		0			
6.8	120.0										381.06		8,199		14.15	20.95	6.80
		221.1	110.6	− 60.6	(20.92 19.70) + 2605	(14.15 15.01) − 0.0192	− 1.9	− 0.0233	− 0.0425	44.71		− 2709		− 0.86			
4.9	101.1										425.77		5,490		15.01	19.90	4.89
		186.1	93.1	− 43.1	(19.88 19.25) + 2050	(15.01 15.95) − 0.0417	− 1.4	− 0.0210	− 0.0627	22.33		− 962		− 0.93			
3.5	85.0										448.10		4,528		15.94	19.42	3.48
		157.4	78.7	− 28.7	(19.42 19.15) + 1950	(15.94 16.65) − 0.0400	− 0.9	− 0.0147	− 0.0547	16.45		− 472		− 0.66			
2.6	72.4										464.55		4,056		16.60	19.20	2.60
		134.8	67.4	− 17.4	(19.18 19.06) + 2100	(16.60 17.60) − 0.0500	− 0.6	− 0.0083	− 0.0583	10.29		− 179		− 0.51			
2.0	62.4										474.84		3,877		17.11	19.10	1.99
		103.4	51.7	− 1.7	(19.10 19.00) + 1250	(17.12 18.20) − 0.0517	− 1.0	− 0.0014	− 0.0531	18.83		− 32		− 0.97			
1.0	41.0										493.67		3,845		18.08	19.08	1.00
		68.0	34.0	+ 16.0	(19.10 19.19) + 2260	(18.10 18.70) − 0.0591	− 0.5	+ 0.0071	− 0.0520	9.62		+ 154		− 0.57			
0.5	27.0										503.29		3,999		18.65	19.15	0.50
		27.0	13.5	+ 36.5	(19.17 19.40) + 19.70	(18.67 19.35) − 0.0730	− 0.5	+ 0.0185	− 0.0545	9.17		+ 335		− 0.67			
0	0										512.46		4,334		19.32	19.32	0

The correct value of E_2 corresponding to E_1 = 23.95 ft as obtained by Step 1 is E_2 = 19.30 ft. The value E_1 = 23.95 ft is used here as an approximation to correspond to a value of A_p = area of one 48" CMP (see Step 3). The three values E_1 = 23.95 ft, E_2 = 19.30 ft, and A_p = area of 48" CMP are a set of values which are all to be in correspondence. Since A_p has been approximated a routing for 48" CMP is performed in Step 4 to check this correspondence. By the results of routing, the value of E_2 is found when E_1 = 23.95 ft and A_p = area of 48" pipe. This value is E_2 = 19.32 ft. Since E_1 = 23.95 ft and E_2 = 19.30 ft are the correct corresponding values, the results of Step 4 dictate that the correct value of the corresponding A_p is slightly larger than the area of a 48" CMP.